Tribhuvan University

Institute of Science and Technology

BSc. CSIT, Third Semester

Course Title: Numerical Method

Course No.: CSC 204 **Full Marks:** 60+20+20

Credit Hours: 3 **Pass Marks:** 24+8+8

Nature of Course: Theory (3hrs.) + Lab (3hrs.)

Course Synopsis: This course contains the concept of numerical techniques of solving the differential equations and algebraic equations.

Goal: To be familiar with the theory of numerical analysis for solving algebraic equations, solution of ordinary and partial differential equations related to engineering, computer science and mathematics.

Course Contents:

Unit 1: Solution of Nonlinear Equations (10 hrs.)

1.1 Review of Calculus and Taylor's Theorem– Definition of Big O notation, statement of Taylor's Theorem, meaning of solution of non-linear equations.
1.2 Errors in numerical Calculations – Truncation, round-off, errors in original data, blunders, propagated error and floating point arithmetic, errors in converting values, relative-absolute errors.
1.3 Trial and Error Method
1.4 Half-Interval Method – the bisection method, algorithm, implementation and convergence.
1.5 Secant Method – secant method, algorithm, implementation and convergence
1.6 Newton's Method – the Newton's method, algorithm, relating Newton's method to other method, complex roots, implementation and convergence.
1.7 Fixed-point Iteration – the method, different rearrangements, the algorithm, implementation and convergence.
1.8 Newton's Method for Polynomials – the method, synthetic division algorithm and remainder theorem, Horner's method and algorithm, implementation and convergence.

(Relations between different methods, their advantages, disadvantages and comparison between different tests are essential)

Unit 2: Interpolation and Approximation (8 hrs.)

2.1 Interpolation – definition, application, definition of extrapolation.

2.2 Lagrange's Interpolation – Lagrangian polynomials, errors, algorithms, numerical applications and implementations.

2.3 Newton's Interpolation – divided difference, algorithm for constructing divided difference table, divided difference for a polynomial, error of interpolation, evenly spaced data, differences vs. divided differences, algorithms and implementations.

2.4 Cubic Spline Interpolation – definitions, derivation, algorithm, examples illustrating cubic spline interpolation.

2.5 Least Squares Approximation – definition and application, derivations, algorithm and implementation of least square approximation for linear and non-linear exponential and polynomial data.

(Each method should be implemented and compared with each other drawing conclusions of their advantages, accuracy and error)

Unit 3: Numerical Differentiation and Integration (5 hrs.)

3.1 Numerical Differentiation – definition, application, Derivatives from divided difference table, error term, algorithm, Derivatives for evenly spaced data, forward difference formula, central difference formula, error terms, Second order derivatives, Maxima and Minima of tabulated function.

3.2 Numerical Integration – definition, application, Newton-Cotes quadrature formulas, trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule, Composite formulas for trapezoidal & Simpson's rules, algorithms and implementation, Gaussian integration algorithm, derivation, Romberg integration formula, algorithm.

Unit 4: Solution of Linear Algebraic Equations (10 hrs.)

4.1 Review – existence and uniqueness of solution of systems of linear equations, properties of matrices.

4.2 Gaussian elimination method and algorithm, pivoting, ill-conditioning, Gauss-Jordan method and algorithm, matrix inversion.

4.3 Matrix Factorization – Dolittle algorithm, Cholesky's factorization.

4.4 Iterative Methods – Jacobi method and Gauss-Seidel method

4.5 Eigen values and eigen vectors problems, solving eigen value problems using power method.

Unit 5: Solution of Ordinary Differential Equations (7 hrs.)

5.1 Review of differential equations, definition of ordinary differential equations and examples, order and degree, initial value problems.

5.2 Taylor's series method and error term, Picard's method, Euler's method and its accuracy, Heun's method, Runge-Kutta methods (4^{th} order method: formula and problem solution), algorithms and implementation.

5.3 Solutions of higher-order equations – definitions and examples of higher order difference equations, solution of system of differential equations, representation of higher order equations into system of equations.

5.4 Boundary Value Problems – definitions and examples, shooting method and its algorithm.

Unit 6: Solution of Partial Differential Equations (5 hrs.)

Review of partial differential equations, deriving difference equations, Laplacian equation and Poisson's equations, their solution techniques, algorithms and examples.

References:

1. C.F. Gerald and P.O. Wheatley, Applied Numerical Analysis, Addison-Wesley Publishing Company, New York.
2. E. Balagurusamy, Numerical Methods, Tata McGraw-Hill.
3. W. Cheney and D. Kincaid, Numerical Mathematics and Computing, Brooks Cole Publishing Co.
4. S. Yakowitz and F. Szidarovszky, An Introduction to Numerical Computations, Macmillan Publishing Co, New York.
5. W. H. Press, B.P. Flannery et al, Numerical Recipes in C, Cambridge Press.

Laboratory Works:

This laboratory experiments will consists of program development and testing of non-linear equations, interpolation, numerical integration, and differentiation, linear algebraic equations, and differential equations. Each algorithm should be implemented in an appropriate language (C, C++). However, software like Maltab-Mathematica and Octave are also recommended for the supplements.

List of Lab Exercises:

1. Solution of non-linear equations using Bisection method and Secant method.
2. Solution of non-linear equations using Newton-Raphson method and Fixed-point iteration method.
3. Solution of polynomial using Newton's method, use Horner's rule to evaluate polynomial.
4. Polynomial interpolation using Lagrange's interpolation and Newton's Divided Difference Interpolation.
5. Fitting a linear and non-linear function using least square method
   i. Linear (straight line) i.e. $y = ax + b$
   ii. Non-linear functions
      a) Exponential function i.e. $y = ae^{bx}$
      b) Quadratic function i.e. $y = ax^2 + bx + c$
6. Derivative from divided difference table.

7. Integration using Trapezoidal rule, Simpson's 1/3 rule and Simpson's 3/8 rule.
8. Solution of system of linear equations using Gauss Elimination method (using partial pivoting)
9. Gauss-Jordan Method.
10. Gauss-Seidel method, Power method.
11. Solution of ordinary differential equation using Euler's method. Heun's method and 4^{th} Order Runge-Kutta method.
12. Boundary value problems using shooting method.

Remarks:

1. This detail course is based on the prescribed course of study. The respective teacher should follow the prescribed course of BSc. CSIT for the teaching.
2. Following teachers from following colleges have been involved to prepare this document.

Name of the Campus	Name of Teacher
CDCSIT, TU	Dr. Tanka Nath Dhamala
Patan Multiple Campus	Sharad Kumar Ghimire Modick Bahadur Basnet
Amrit Science Campus	Urmila Pyakurel
Mahendranagar	----------
New Summit	Urmila Pyakurel
St.Xavier's Campus	Rajiv Narkarmi
Model College of Management, Janakapur	Dipendra Kumar Jha
Kathford Int'l College	Rajesh Prasad Shrestha Urmila Pyakurel Ram Kaji Budhathoki