Tribhuvan University
Institute of Science and Technology
Bachelor of Science in Computer Science and Information Technology
Detailed Syllabus

Course Title: Linear Algebra
Course No.: MTH-155 **Full Marks: 80+20**
Credit Hours: 3 **Pass Marks: 32+8**
Nature of Course: Theory
Course Synopsis: Linear equations in linear algebra, matrix algebra, determinants, vector spaces, eigenvalues and eigenvectors, orthogonality and least squares.

Goal: This course provides students with the knowledge of fundamental of linear algebra and the theory of matrices. On completion of this course the student will master the basic concepts and acquires skills in solving problems in linear algebra.

Course Contents:

Unit 1: Linear Equations in Linear Algebra **(10 hrs.)**

1.1 Systems of Linear Equations

Linear equation with real or complex coefficient, systems of linear equations, solution to the system (no solution, exactly one solution or infinitely many solutions), definitions of consistent and inconsistent systems, matrix notation of the system, definition of augmented matrix, solving the linear system by elementary row operations, definition of row equivalent matrices, existence and uniqueness questions, numerical note on floating point arithmetic and partial pivoting, related examples.

1.2 Row Reduction and Echelon Forms

Definitions of echelon form (row echelon form), reduced echelon form (reduced row echelon form), echelon matrix (reduced echelon matrix), row reduced matrix, statement of Theorem 1 (each matrix is row equivalent to one and only one reduced echelon matrix), definition of pivot position and pivot column, the row reduction algorithm, identification of forward phase and backward phase of the algorithm. Solutions of linear systems: definitions of basic and free variables, general solution of linear system, back-substitution, idea of arithmetic operations for solving the system, statement of Theorem 2, solution of a system by row reduction, examples to illustrate the algorithm.

1.3 Vector Equations

Vectors in R^2 and geometry vectors in R^n , linear combinations and weights, definition of the subset of R^n spanned by vectors, geometry of span {v} and span{n, r}, related problems.

1.4 The Matrix Equations Ax=b

Definition of Ax, Statement of Theorem 3, existence of solutions (the equation $Ax=b$ has a solution if and only if b is a linear combination of the columns of A), Statement of Theorem 4, verification of Theorem 5, related exercises.

1.5 Solution Sets of Linear Systems

Homogenous linear systems, trivial and nontrivial solutions, parametric vector equations, solution in parametric vector form, solutions of non-homogeneous systems, translated solution, Statement of Theorem 6, related exercises

1.6 Linear Independence

Definition of linear independent and linear dependent vectors, linear independence of matrix columns, characterization of linearly dependent sets (one or two vectors, two or more vectors), proofs of Theorem 8 and 9, related exercises.

1.7 Introduction to Linear Transformations

Definition of transformation/function/mapping, domain, co-domain of a transformation, image and range of a vector, matrix transformations, shear transformation, contraction, dilation, related exercises.

1.8 The Matrix of a Linear Transformations

Proof of Theorem 10, definition of the standard matrix for the linear transformation T, geometric linear transformations of $\boldsymbol{R}^2$, existence and uniqueness questions, (Tables 1-4, page 103) definition of one-to-one, onto mapping $\boldsymbol{T}: \boldsymbol{R}^n \to \boldsymbol{R}^m$, Proofs of Theorems 11, 12, related exercises.

Unit 2: Matrix Algebra

2.1 Matrix Operations

Sums and scalar multiples, matrix multiplication, proofs of Theorem 2 and 3, related examples.

2.2 The Inverse of a Matrix

Definition of matrix inverse, singular and non-singular matrices, proofs of Theorems 4,5 and 6 and statement of theorem 7, an algorithm for finding the inverse of a matrix A, examples illustrating the results and algorithm.

2.3 Characterizations of Invertible Matrices

Statement of Invertible matrix theorem, invertible linear transformations, proof of Theorem 9, ill-conditioned matrix concept, related exercises

2.4 Partitioned Matrices

Definitions of partitioned or block matrix, addition scalar multiplication and multiplication of matrices, column-row expansion of AB, inverse of partitioned matrices, numerical importance of partitioning, related exercises.

2.5 The Leontief Input Output Model

Definitions of production vector, final demand vector, intermediate demand, consumption matrix, statement of Theorem 11, column sum, a formula for $(I - C)^{-1}$ and economic importance of entries in it, numerical importance, related exercises

2.6 Applications to Computer Graphics

Introduction, use of shear transformation, homogeneous coordinates in 2D, coordinates in 3D, projection maps, center of projection, related exercises

Unit 3: Determinants

3.1 Introduction to Determinants

Determinant of n×n matrix A, Statement of Theorem 1, 2, numerical drawback of the direct calculation of the determinant, related examples

3.2 Properties of Determinants

Row operations, statement of Theorem 3, proof of Theorem 4, numerical importance of determinant calculation by row operation, column operations, determinants and matrix products, linearity property of the determinant function, statement of Theorem 6, related problems.

3.3 Cramer's Rule, Volume and Linear Transformations

Prof of Cramer's rule, proof of inverse of a matrix (Theorem 8), numerical note of Cramer's rule, determinants as area or volume, linear transformation, applications of Theorems 9 and 10, related exercises

Unit 4: Vector Spaces

4.1 Vector Spaces and Subspaces

Definition of vector space, examples of vector spaces (examples 1-5: page 233-245), definition of a subspace, examples (examples 6-10: page 236-237), statement of Theorem 1, related exercises

4.2 Null Spaces, Column Spaces and Linear Transformations

Definition of null space, column space, proofs of Theorem 2,3 examples, relation between the null space and column space, Kernel and range of a linear transformation (definition and examples), related problems.

4.3 Linearly Independent Sets, Bases

Definitions of linearly dependent and independent vectors, examples, proof of Theorem 4, definition of basis and standard basis, statement of the spanning set theorem, (Theorem 5), bases for null and column spaces, statement of Theorem 6, related exercises.

4.4 Co-ordinate Systems

Proof of unique representation theorem (theorem 7), coordinate of a vector with respect to the basis, a graphical interpretation of coordinates, change of coordinates matrix, proof of Theorem 8, isomorphism form a vector space onto another vector space, related exercise.

4.5 The Dimension of a Vector Space

Proof of Theorem 9, 10, finite/infinite-dimensional vector spaces and dimension of a vector space, statement of basis theorem (theorem 12), subspace of a finite-dimensional space; the dimensions Null A and Col A, related exercises.

4.6 Rank

Definition of row space, statement of Theorem 13, definition of rank, statement of rank theorem, statement of invertible matrix theorem, related exercises.

4.7 Change of Basis

Idea of change of basis, change of co-ordinate matrix (statement of Theorem 15), change of basis in R^n, related exercises

Unit 5: Eigenvalues and Eigenvectors

5.1 Eigenvectors and Eigenvalues

Definition of Eigen vector and Eigen values, proofs of Theorem 1 and 2. Eigen vectors and difference equation, related exercises

5.2 The Characteristic Equation

Definition of characteristics equation, statement of the invertible matrix theorem with respect to Eigen value, calculation of Eigen value and Eigen vector, definition of similarity transformation, proof of Theorem 4, related exercises.

5.3 Diagonalization

Definition of a diagonalizable matrix, Eigen vector basis, statement of Theorem 5, diagonalizing matrices algorithm, proof of Theorem 6, application of Theorem 7, related exercises.

5.4 Eigenvectors and Linear Transformations

The matrix of linear transformation, linear transformation from V to V, linear transformation on R^n proof of Theorem 8, similarity of matrix representations, numerical note, related exercises.

5.5 Complex Eigenvalues

Definition of complex Eigen values and complex eigenvectors, real and imaginary parts of vectors, eigenvectors and eigenvalues of a real matrix that acts on C^n, application of Theorem 9, related problems.

5.6 Discrete Dynamical System

Motivation, a predator-prey system, graphical description of solutions, change of variable, complex eigenvalues.

Unit 6: Orthogonality and Least Orthogonality

6.1 Inner Product, Length and Orthogonality

Definition of inner (dot) product, properties of dot product, the length of a vector, unit vector, normalizing distance between two vectors, orthogonality of vectors, proof of the Pythagorean Theorem 2, orthogonal components, statement of Theorem 3, angle between the vectors in R^2 and R^3, related exercises.

6.2 **Orthogonal** Sets

Definition of orthogonal set, proof of Theorem 4, orthogonal basis, proof of Theorem 5, and its geometry, orthogonal projection, orthogonal sets, orthonormal basis, proofs of Theorem 6,7 geometry, orthogonal projection, orthonormal sets, orthonormal basis, proofs of Theorems 6,7, related exercises.

6.3 Orthogonal Projections

Definitions, statement of orthogonal decomposition theorem, proof of best approximation theorem, statement of Theorem 10, related exercises.

6.4 The Gram-Schmidt Process

The process of Gram-Schmidt, statement of Theorem 11, orthonormal bases, the algorithm QR-factorization, related exercises.

6.5 Least-Squares Problems

Definition of a least-squares, solution of Ax=b, solution of the general least-squares problems; proof of Theorem 13, application of Theorem 14,15, related exercises.

6.6 Applications to Linear Models

Definition of least-squares lines, illustration of the method, the general linear model, least squares fitting of curves, related exercises.

Text Book:

David C. Lay: Linear Algebra and its Application, 3rd edition, Pearson Education.

Reference Book

1. Bernard Kolman: Introductory Linear Algebra with Application 7th edition, Pearson
2. Gilbert Stang: Linear Algebra and its Application 3rd edition.

3. E. Kreszig: Advanced Engineering Mathematics 5th edition, Wiley.

Remarks

1. Theory and practice should be done side by side
2. Theory classes 4hrs and tutorial classes 2 hrs per week
3. Recommended to use Mathematica/Matlab/Maple for testing selected exercises.