Micro-Syllabus
Bachelor of Science in Computer Science and Information Technology
Institute of Science and Technology
Tribhuvan University

Course Title: Calculus and Analytical Geometry
Course no: MTH-104 **Full Marks:** 60+20+20
Credit hours: 3 **Pass Marks:** 24+8+8
Nature of course: Theory
Course Synopsis: Preliminary revisions of differentiation and integration; techniques of integration; infinite series; vectors and analytical geometry in space (differential geometry); vector valued functions; multivariable functions and partial derivatives.

Goal: This course aims at providing students with some advanced topics in undergraduate calculus and fundamental concepts of partial differentiation and P.D.E. of second order. It is assumed that a student who has Certificate Level papers in mathematics will be able to study this course.

Unit 1. Topics in Differential and Integral Calculus

1.1 Functions and graphs

Cartesian products, relations, graph of relations, definition of a function, domain and range of a function, composite functions, one-to-one and onto functions, even and odd functions, piecewise defined functions, graphs of functions, examples.

1.2 Extreme values of functions, graphing of derivatives

Limits: Formal definition of limits (one-sided and two-sided), verifications.

Theorem: A function $f(x)$ has a limit as x approaches to c if and only if it has left-hand and right hand limits there and these one sided limits are equal (**without proof**).

Continuity: Definitions of continuity at interior point, at end points and on intervals of domain with examples.

Theorem (The Intermediate Value Theorem): Suppose $f(x)$ is continuous on an interval I, and a and b are any two points of I. Then, if y_o is a number between $f(a)$ and $f(b)$, there exists a number c between a and b such that $f(c)= y_o$ (**without proof**).

Derivatives: Definitions (one-sided and two-sided) with examples of differentiable and non-differentiable functions, instantaneous rate of change and geometrical meaning.

Theorem: A function has a derivative at a point if and only if it has left-hand and right-hand derivatives there and they are equal (**without proof**).

Theorem: If f has derivative at a point $x = c$, then it is continuous at $x = c$. Then converse does not hold.

Proof of the Theorem.

Applications of derivatives: Definition of global and local minima and maxima, critical point.

Theorem (The Max-Min Theorem for Continuous Functions): If f is continuous at every point of a closed interval I, then f assumes both an absolute maximum and an absolute minimum value somewhere in I (**without proof**).

Counter examples: The requirements that the interval *I* be closed and *f* be continuous play key role in the sense that the **Theorem** need not hold without them.

Theorem (The First Derivative Theorem for Local Extreme Values): If *f* has a local maximum or minimum value at an interior point *c*, then *f'*(c) = 0 (**without proof**).
Definition of increasing and decreasing functions, examples.
Theorem: The First Derivative Test for increasing and decreasing functions (**only statement**).
The first derivative test for local extreme values (at critical points and at end points), definition of concavity, the second derivative test for concavity, points of infection, the second derivative test for local extreme values, graphing of y' and y'' (page231), related examples and exercises in each test and graphing.

1.3 Mean value Theorems

Theorem (Rolle's Theorem): Suppose $y = f(x)$ is continuous on a closed interval $[a, b]$ and differentiable at every point of its interior (a, b). If *f*(a) = *f*(b) = 0, then there is at least one number c ∈ (a, b) such that *f'*(c) = 0 .
Proof of the Theorem, counter examples to show that the assumptions of the theorem are essential, examples to illustrate the points the theorem yields.
Theorem (The Mean Value Theorem): suppose $y = f(x)$ is continuous on a closed interval $[a, b]$and differentiable on the open interval(a, b). Then there is at least one point c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.
Proof of the Theorem, examples to illustrate the points the theorem yields.
Corollary: Functions with zero derivatives are constant.
Corollary: Functions with the same derivative differ by a constant.
Corollary: Suppose that f is constant on $[a, b]$ and differentiable on (a, b). Then f increases on $[a, b]$ if $f' > 0$ at each point of (a, b). Likewise, f decreases on $[a, b]$ if $f' < 0$ at each point of (a, b).
Proofs of all corollaries and examples to illustrate the results.

1.4 Definite integrals, properties and application, Mean Value Theory for definite integrals

Definite Integrals: Definition of Riemann sum for a function f on the interval $[a, b]$, the definite integral as a limit of Riemann sums.
Theorem: All continuous functions are integrable (**only statement**).
Examples to illustrate some discontinuous functions may be integrable, integration of a constant function by definition.
Definition: Let $f(x) \geq 0$ be continuous on $[a, b]$. The area of the region between the graph of f and the x-axis is $A = \int_a^b f(x)dx$.
Examples related to the theorem.
Properties of definite integrals: Rules for definite integrals without proof (zero, order of integration, constant multiplies, sums, differences, additivity, dominance), verification of each property with examples.

Mean Value Theory for Definite Integrals:

Theorem (Mean Value Theorem for Definite Integrals): If f is continuous on $[a, b]$, then at some point c in $[a, b]$, $f(c) = \frac{1}{b-a}\int_a^b f(x)dx$.

Proof of the Theorem, examples.

Applications: Integrals and total area, average value of an arbitrary continuous function with examples.

1.5 Fundamental Theory of Integral Calculus and Application, Improper Integrals

Fundamental Theorems: Parts 1 and 2 (**only statements**), and verifications.

Theorem: If f is continuous on $[a, b]$ then $F(x) = \int_a^x f(t)dt$ has a derivative at every point of $[a, b]$ and $\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt = f(x), x \in [a, b]$.

Theorem: If f is continuous at every point of $[a, b]$ and F is any antiderivative of f on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$.

Applications: Areas between the curves, integration with respect to x and with respect to y, lengths of place curves $y = f(x)$ and $x = g(y)$.

Improper Integrals: Definitions 1-4 (page 613) where f is continuous on $[a, \infty)$ and $(-\infty, b]$, $(a, b]$ and $[a, b)$ with examples in each case, convergence and divergence of improper integrals.

Definition: If f becomes infinite at an interior point d of $[a, b]$, then

$$\int_a^d f(x)dx = \int_a^d f(x)dx + \int_d^b f(x)dx.$$

Examples related to the definition. Integrals from $-\infty$ to ∞, convergence-divergence and verifications of simple functions.

Unit 2. Infinite Series

2.1 Infinite sequence and series of convergence and divergence

Definitions (infinite sequence, convergence and divergence of infinite sequence, bounded non decreasing sequence and least upper bound, infinite series, sequence of partial sums, convergence and divergence of the series), geometric series, telescoping series, the n^{th} term test for divergence, combining series (rules: sum, difference, constant multiple **without proofs**), adding or deleting terms to a series, re-indexing, examples in each case.

Theorem (non-decreasing sequence theorem): A non-decreasing sequence of real numbers converges if and only if it is bounded from above. If a non-decreasing sequence converges, it converges to its least upper bounded (**only statement**)

2.2 Integral Test, Comparison Test, Ratio and Root Test

Theorem: A series $\sum_{n=1}^{\infty} a_n$ of nonnegative terms converges if and only if its partial sums are bounded from above.

Integral Test: Divergence of the harmonic series $\sum_{n=1}^{\infty}\frac{1}{n}$, convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$, test of the series $\sum_{n=1}^{\infty}\frac{1}{n^p}$, statement of the integral test (pages 660), related problems.

Comparison Test: Convergence of the series Comparison test $\sum_{n=1}^{\infty}\frac{1}{n!}$, proof of the direct comparison test for the series of nonnegative term (pages663) and the statement of limit comparison test (page 665), related problems.

Ratio and Root Tests: Statements of the ratio test (page668) and nth root test (page671), related problems.

2.3 Absolute and Conditional Convergence

Definitions (Alternating Series and Alternating Harmonic Series, absolutely convergent, conditionally convergent), statement of **Leibniz's Theorem 8** (page 673), the alternating series test, convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$, proof of absolute convergence test (**Theorem 10**, page 676), statement of **Theorem 11** (page 677), **The Rearrangement Theorem** for absolutely convergence series, related problems in each case.

2.4 Power series, Taylor and Maclaurin Series, Convergence of Taylor Series

Power Series: Definition of power series about $x = 0$ and about$x = a$ and examples, statement of the convergence theorem for power series, the radius and interval of convergence of power series, statements of the term-by-term differentiation and integration theorems, examples.

Taylor and Maclaurin Series: Series representations, definitions of Taylor series, Maclaurin series and Taylor polynomial generated by function f, statement **Taylor's Theorem**, Taylor's formula and Error Term, the Maclaurin series of the type e^x, Sin x, Cos x , Cos $2x$, x Sin x, convergence of Taylor's series (statement of the **Remainder Estimate Theorem**, page 698), related examples in each case.

Unit 3. Conic Section

3.1 Classifying conic sections by eccentricity

Definitions (circle, parabola, ellipse, hyperbola and related terms), examples to explain the defined terms, equations and graphs of the conic sections defined above, classifying the defined conic sections by eccentricity, related problems.

3.2 Plane curves, parametric and polar equations, integration in polar coordinates

Plane curves, parametric and polar equations: Definition $x = f(t), y = g(t)$, examples: circle, semicircle, parabola (half and entire), ellipse, hyperbola, definition of polar coordinates (pole, initial ray, polar pair (r, θ), negative values of r), polar equations for lines, circles, ellipse, parabolas and hyperbolas, related problems.

Integration in polar coordinates: Area in the plane, area of the fan-shaped region between the origin and the curve, areas enclosed by cardioid, limacon, area of the region $0 \leq r_1(\theta) \leq r \leq r_2(\theta)$, length of a curve, related problems.

Unit 4. Vectors and vectors Valued Functions

4.1 Vectors in the space

Vectors in the space, algebra of vectors in space, length, unit vectors, zero vector, distance between two points, scalar product, derivation of dot product $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 - a_2b_2 + a_3 b_3$, laws of vector addition, cross product, perpendicular and parallel vectors, the associative and distributive laws, determinant formula for cross product, $\boldsymbol{a} \times \boldsymbol{b}$ as the area of a parallelogram, examples.

4.2 Lines and planes in space

Vector equation for the line through a given point and parallel to a vector, standard parameterization of the line through a point and parallel to a vector, the distance from a point to a line in a space, equation for planes in space, plane through a point normal to n. A plane determined by three points, intersection of a line and a plane, the distance from a point to a plane, lines of intersection, related problems.

4.3 Cylinders and quadratic surfaces

Definition of cylinder, generating curve, and examples of cylinders, definition of quadratic surface (ellipsoid, paraboloid).

4.4 Cylindrical and spherical coordinates

Definition of cylindrical coordinates, equations in (x, y, z) and (r, θ, z) coordinates, definition of spherical coordinates, equations in spherical and cylindrical coordinates, examples in both cases.

4.5 Vector valued functions and space curves

Definitions (curve, path, position vector, composite functions, vector function, scalar function), the curve Helix, definitions of limit, continuity and derivative, rules of differentiation for vector valued functions (without proof), integrals of vector valued functions, definite integrals, examples in each case.

4.6 Unit tangent vector, curvature and torsion and TNB system

Definitions (unit tangent vector, curvature, principal unit normal, radius of curvature, torsion, TNB system), examples in each case.

Unit 5. Multiple Integrals

5.1 Double integrals in rectangular coordinates

Double integral formula over rectangles, properties of double integrals (without proof), double integral as volumes, **Fubini's Theorem for calculating double integrals** (only statements), finding limits of integration, related problems in each case.

5.2 Finding areas, moments and centers of mass

Areas of bounded regions in the plane (definition and examples), first and second moments and center of mass (definition and examples), centroid of geometric figures, double integrals in polar coordinates, limits of integration in polar coordinates, area in polar coordinates, changing Cartesian integrals into polar integrals, related problems.

5.3 Triple integrals in rectangular coordinates and application

Definition of triple integrals, properties of triple integrals (without proof), volume of a region in space, limit integration in triple integrals, mass and moment formulas for objects in space, related problems in each case.

5.4 Substitutes in multiple integrals

Definition of Jacobian determinant, substitutions in double and triple integrals (transformations and examples), related problems.

Unit6. Multivariate Calculus

6.1 Functions, limits and continuity of two or more variables

Definitions (functions, limits, continuity of functions of two or more variables), examples in each case.

6.2 Partial derivatives

Definition of partial derivatives with respect to all variables, calculation of partial derivatives, second order partial derivatives, **Euler's Theorem** (without proof), examples for the verification.

6.3 Differentiability, differentials, total differential coefficients.

Definition of differentiability, statement of the **Increment Theorem** for functions of two variables.

Theorem: If a function f(x,y) is differentiable at (x_0,y_0) , then it is continuous at this point (without proof).

Linearize a function of two variables, standard lines approximation, the error in the standard linear approximation, predicting change with differentials, total differential coefficients, the chain rule for functions of two and three variables, implicit differentiation, related problem.

6.4 Directional derivative and gradient vectors

Directional derivative in the plane (definition, geometrical interpretation, properties and examples), definition of the gradient vector, related problems.

6.5 Extreme values

Definitions of local maxima and local minima, **Theorem 7 (**only statement), page 984, (first derivative test for local extreme values), definitions and equations for tangent planes and normal lines, definitions of critical point and saddle point, **Theorem 8** (only statement), page 990, (Second derivative test for local extreme values), absolute maxima and minima on closed bounded regions, summary of max-min tests, related problems.

6.6 Lagrange multipliers

Constrained maxima and minima, statement of orthogonal gradient, **Theorem** and its corollary (page 1001), the method of Lagrangian multipliers with two constraints, related problems.

Unit 7. Partial Differential Equations

7.1 Review of ordinary differential equations

Basic concepts and idea, geometrical meaning, separable ode's, exact ode's liner ode's (Chapter 1, Section 1.1, 1.2, 1.3, 1.5, 1.6), ,linear differential equations of the second order (Chapter 2, Sections 2.1, 2.2, 2.3), examples in each case.

7.2 Analysis of partial differential equations of the first order

Definition, origins of pde's, definitions of complete, general, singular and particular integrals, related problems.

7.3 Linear pde's of the first order

Review of method of solution of differential equation dx/P = dy/Q = dz/R, the general solution of the linear partial differential equation Pp + Qq = R (statement of **Theorem2,** page 50, I.N. Sneddon), geometrical meaning, problem (finding the general solutions of the linear pde's).

7.4 Partial differential equations of the second order

Definition, origins, basic concepts of second order pde's, derivation of second order pde's, related examples.

7.5 Solutions of general partial differential equations

Linear pde's with constant coefficients, definitions of complimentary and particular solutions, definitions of reducible and irreducible equations, implementations of **Theorems 1-9** by solving related problems (page 97-102, I.N.Sneddon).

7.6 Wave equations and heat equations and their solutions

Basic concepts of wave and heat equations, boundary and initial conditions of heat and wave equations, mathematical modeling of vibrating string, D'Alembert's solutions of wave equation, solution of heat equation by Fourier series, related examples (E.Kreyszig, Chapter 11, Section 11.1, 11.2, 11.4, 11.5).

Text Books:

1. G.B. Thomas and R.L. Finney, Calculus and Analytical Geometry, 9^{th} Edition, Pearson Education Pvt. Ltd.
2. E. Kreyszing, Advanced Engineering Mathematics, John – Wiley & Sons, 8^{th} Edition.

References:

1. E. W. Swokowsli, Calculus with Analytical Geometry, Second Alter Edition.
1. I. Snedden, Elements of partial Differntial Equation, McGraw-Hill Book Company.

Remarks:

1. Theory and practice should be done side by side.
2. It is recommended to use either Mathematica or Maple or Matlab for implementation of selected examples.
3. There should be FOUR hours for theory class and additional TWO hours for tutorial class every week during the semester for this course.