1. Course Contents

Mathematics I

Course Title: Mathematics I (Calculus) Full Marks: 100
Course No: MTH 112 Pass Marks: 40
Nature of the course: Theory and Practice Credit Hrs.: 3

1. Unit 1: Function of One Variable: Four ways of representing a function, Linear mathematical model , Polynomial, Rational, Trigonometric, Exponential and Logarithmic functions. Combination of functions, Range and domain of functions and their Graphs. **5 Hours.**

2. Unit 2: Limits and Continuity: Precise definition of Limit, Limits at infinity, Continuity, Horizontal asymptotes, Vertical and Slant asymptotes. **4 Hours.**

3. Unit 3: Derivatives: Tangents and velocity, Rate of change, Review of derivative, Differentiability of a function, Mean value theorem, Indeterminate forms and LHospital rule. **4 Hours.**

4. Unit 4: Applications of Derivatives: Curve sketching, Review of maxima and minima of one variable, Optimization problems, Newtons method.**4 Hours.**

5. Unit 5: Antiderivatives: Review of antiderivatives, Rectilinear motion, Indefinite integrals and Net change, Definite integral, The Fundamental theorem of calculus, Improper integrals. **5 Hours.**

6. Unit 6: Applications of Antiderivatives: Areas between the curves, Volumes of cylindrical cells, Approximate Integrations, Arc length, Area of surface of revolution. **5 Hours.**

7. Unit 7: Ordinary Differential Equations: Introduction, Introduction to first order equations Separable equations, Linear equations, Second order linear differential equations, Non homogeneous linear equations, Method of undetermined coefficients. **6 Hours.**

8. Unit 8: Infinite Sequence and Series: Infinite sequence and series, Convergence tests and power series, Taylors and Maclaurins series. **5 Hours.**

9. Unit 9: Plane and Space Vectors: Introduction, Applications, Dot product and cross Product, Equations of lines and Planes, Derivative and integrals of vector functions, Arc length and curvature, Normal and binormal vectors, Motion in space. **5 Hours.**

10. Unit 10: Partial Derivatives and Multiple Integrals: Limit and continuity, Partial derivatives, Tangent planes, Maximum and minimum values, Multiple integrals. **3 Hours.**

Text Book
Calculus Early Transcendentals, James Stewart, 7E, CENGAGE Learning.
Reference Book
Calculus Early Transcendentals, Thomas, 12th Editions, Addision Wesley.

Detail Course, 2074/09/14-15
Unit 1:
Basic Concepts, a revision: 3 Hrs.

- Diagonastic Tests Algebra: 8, 9, 10.
- Diagonastic Tests Geometry: 4, 5.
- Diagonastic Tests Calculus: All Problems.
- A Preview of Calculus: Meaning of, area, tangent, velocity and limit of a sequence.

1. Function of One Variable: 5 Hrs.
Four ways of representing a function:
Verbally (by a description in words),
numerically (by a table of values),
visually (by a graph),
algebraically (by an explicit formula).
All workout examples of this section.
Piecewise function, Symmetry , increasing and decreasing function
Exercises 1.1: 3,4, 7,8,23,24, 39, 43, 45, 47, 49,
Linear mathematical model,
Polynomial, Rational, Trigonometric,
Exponential and Logarithmic functions.
Combination of functions, Range and domain of functions and their Graphs.
Exercises 1.2: $10, 11, 12$ and all workedout examples of 1.3.
Give some idea to plot graph using the program like Desmos.
Review of inverse of functions.
Exercises 1.2: $31, 36, 40, 41, 46, 47$
1.5: All worked out examples.

Unit 2:
Limit: Ideas of limits with a calculator (we may use Desmos to get idea of limit from graphs)
The limit of a function: All worked out examples.
Exercises 2.2: $29, 31, 35, 38$
2.3, 2.4, 2,5, 2.6: Worked out examples
Exercises 2.6: $41, 42, 43, 44, 45$

Unit 3:
Derivatives and Rate of Change Worked out examples

Exercises 2.7: $27, 32, 43$
2.3, 2.4, 2,5, 2.6: Worked out examples
2.8: Worked out examples
4.8 Mean Value Theorem: Worked out examples
Exercises 4.8: $1 - 18$ and related problems.
4.4 Indeterminate Forms and L'Hospital's Rule: All worked out examples. **Exercises 4.4:** Related problems.

Unit 4: Curve Sketching: Guide lines for curve sketching and all worked out examples.
Exercises 4.5: $1 - 20$ and some relted problems. Also, we can try with calculator using like Desmos
4.7 Optimization Problems: All worked out examples and related problems.
4.8 Newton's Method: All worked out examples and related problems.

Unit 5:
Review of antiderivatives, Areas and distance, Rectilinear motion: Worked out examples.
5.1 Areas and distance Worked out examples.
5.2 Definite Integrals: Worked out examples.
5.3 Fundamental Theorem of Calculus: Worked out examples.
5.4 indefinite Integrals and the Net Change Theorem: Worked out examples.
7.8 Improper Integrals: Worked out examples.

Unit 6:
6.1 Areas Between the curves: Worked out examples and related problems.
6.2 Volumes: Worked out examples
6.3 Volumes by cylindrical shells: Worked out examples and related problems.
7.7 Approximate Integration: Trapezoidal rule, Simpson Rule, Worked out examples and related problems.
8.1 Arc Length: Worked out examples and related problems.
8.2 Area of a surface of Revolution: Worked out examples and related problems.

Unit 7:
Introduction to the differential equations: Definition, order, degree and model (9.1)
Exercises: 9.1: 1-8
9.3 Separable Equations, Orthogonal trajectories: Worked out examples and related problems.
9.5 Linear Equations: Worked out examples and related problems.
17.1 Second Order Linear ODEs: Worked out examples and related problems.
17.2 Non homogeneous linear ODEs: Worked out examples and related problems.
Method of Undetermined Coefficients: Worked out examples and related problems.

Unit 8:
11.1 Infinite sequence: Worked out examples and related problems.
11.2 Series: Worked out examples and related problems.

Integral test, comparison tests,absolute convergence and ratio tests. Worked out examples
11.8 Power series Worked out examples
11.10 Taylors and Maclaurin's Series: Worked out examles.

Unit 9:
12.1 Introduction: Introduction of plane and space vectors, Worked out examples and related problems.
12.2 Vectors: Combining vectors, Components and worked out examples.
Applications: Worked out example and related examples.
12.3 Dot product: Worked out examples.
12.4 Cross Product Worked out examples.
12.5 Equations of lines and Planes Worked out example.
13.2 Derivative and integrals of Vector Functions: Worked out examples.
13.3 Arc length and curvature worked out examples.
Normal and bi -normal vectors Worked out examples.
Motion in space Worked out examples.

Unit 10:
14.2 Limit and continuity: Worked out examples and 5-18 from exercises 14.2
14.3 Partial derivatives Worked out examples and related problems.
14.5 Chain Rule: Worked out examples and 1-2 from exercises 14.5
14.4 Tangent planes Worked out examples and related problems.
14.8 Lagrange Multipliers Worked out examples and related problems
14.7 Maximum and minimum values Worked out examples and related problems
14.8 Lagrange Multipliers Worked out examples and related problems
15. Multiple integrals: Double Integrals over Rectangles, Worked out examples.
15.2 Iterated Integrals Worked out examples and related problems.

Notes to the Question Setter:

1. All units are equally important, so questions must be made from every units with equal marks as far as possible.
2. In group A, a student can answer any three questions selecting among 4 long questions, each carrying 10 marks. These 4 questions should cover all the 10 units as far as possible.
3. In group B, a student can answer any 10 questions selecting among 11 short questions, each carrying 5 marks. Among these 11 questions, 10 are made from each unit and remaining one question may be made from any one of the 10 units.
4. Questions must be creative and should fit to the allocated time.

Model Question

Model Question

Tribhuvan University

Institute of Science and Technology

BScCSIT Level/First Semester — Full Marks: 80
Mathematics[MTH112] — Pass Marks: 32
Calculus — Time 3 Hrs.

Candidates are required to give their answers in their own words as far as practicable.

Group A ($10 \times 3 = 30$)

Attept any **THREE** questions.

1. (a) A function is defined by [5]

$$f(x) = \begin{cases} 1 - x, \text{ if } x \leq -1 \\ x^2, \text{ if } x > -1 \end{cases}$$

Evaluate $f(-3), f(-1)$ and $f(0)$ and sketch the graph.

(b) Prove that the limit $\lim_{x \to 0} \frac{|x|}{x}$ does not exist. [5]

2. (a) Sketch the curve [5]

$$y = \frac{2x^2}{x^2 - 1}$$

(b) Estimate the area between the curve $y = x^2$ and the lines $x = 0$ and $x = 1$, using rectangle method. [5]

3. (a) Show that the volume of a sphere of radius r is $4/3\pi r^3$. [3]

   (b) Define initial value problem. Solve: [3]

$$x^2y'' + xy' = 1, x > 0, y(1) = 2$$

   (c) Find the Maclaurin's series for $\sin x$ and prove that it represents $\sin x$ for all x. [4]

4. (a) Find the curvature of the helix $\mathbf{r}(t) = a\cos t\mathbf{i} + a\sin t\mathbf{j}$. [3]

   (b) If $f(x, y) = xy/(x^2 + y^2)$, does f(x, y) exist, as $(x, y) \to 0$? [3]

   (c) Find the shortest distance from the point $(1, 0, -2)$ to the plane $x+2y+z = 4$ [4]

Group B $(10 \times 5 = 50)$

Attept any **TEN** questions.

5. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2 - x}$, find fog and gog.

6. Define continuity of a function at a point $x = a$. Show that the function $f(x) = 1 - \sqrt{1 - x^2}$ is continuus on the interval $[-1, 1]$.

7. Verify the Rolle's therem for $f(x) = x^3 - x^2 - 6x + 2$ in $[0, 3]$.

8. Find the third approximation x_3 to the root of the equation $f(x) = x^3 - 2x - 5$, setting $x_1 = 2$.

9. Evaluate $\int_0^1 \ln x dx$.

10. Find the volume of the solid obtained by about $y - axis$ the region between $y = x$ and $y = x^2$.

11. Solve: $y'' + y' - 6y = 0, y(0) = 1, y'(0) = 0$

12. Show that the series $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$ converges.

13. If $\mathbf{a} = \mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ find $4\mathbf{a} + 5\mathbf{b}$. Also find the unit vector of $4\mathbf{a} + 5\mathbf{b}$.

14. Find the partial derivative of $f(x, y) = x^3 + x^2y^3 - 2y^2$, at $(2, 1)$.

15. If $z = x^2y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, find $\frac{dz}{dt}$ at $t = 0$.